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1. Introduction

In this paper, we describe the functioning of a multiregional equilibrium search model for the labour market. The essential idea is that regional labour supply is fixed, and that the number of vacancies, the number of unemployed and wage levels are endogenously determined given productivity levels. Due to search frictions, in each region, unemployment and vacancies exist at the same time.

In essence, throughout this paper it is assumed that the number of matches between vacancies and unemployed job seekers in a region depends on the regional levels of vacancies and unemployment. Job seekers search for jobs in their own region but also in other regions. It is assumed that regional labour markets are in equilibrium when the number of individuals which become unemployed in a region are equal to the number of individuals which find employment in this region. Wage levels are endogenously determined. Job seekers and employers bargain about the wage conditional on the commuting costs. When the commuting costs are too high, the job seeker and employer will not form a match. So, the maximum commuting costs are endogenously determined.

The model formulation given here can be considered as a benchmark formulation for an equilibrium model where unemployment and vacancies are an integral part. It is inspired by the pathbreaking work of Pissarides (2000). An important difference is that the Pissarides model is a single regional model. Our contribution is that we derive a formulation in the multiregional context.

2. The Model

2.1 The matching model

We presume that individuals are identical. Firms are identical, but for the level of productivity, which is regional specific. Individuals are either unemployed or employed. The unemployed search for jobs, the employed do not search. A firm consists of only one job, which is either filled or unfilled. Firms do not relocate filled jobs to other regions (due to prohibitive high relocation costs), but may relocate unfilled jobs freely at no costs (so employers are free to choose the location of the vacancy). In order to fill a job, firms post a vacancy. We presume that individuals do not move residence either within or between regions.

We presume that there exist n regions. The labour force of each region contains L_i individuals. We let u_i denote the unemployment rate -- the number of unemployed as a fraction of the labour force in region i -- and v_j , the vacancy rate -- the number of vacant jobs, as a fraction of the labour force in region j (i,j =1,..n).

The unemployed contact each period one vacancy and have a fixed probability of contacting a firm in a region.[1] We denote the probability that an unemployed worker living in region i applies for a vacancy in region j as k_{ij} . Given a large number of unemployed and vacancies, the number of contacts per vacancy in region j can then be approximated by a Poisson process with parameter $k_{ij}\ L_iu_i/v_jL_j$. Now suppose that the firm preselects one applicant from the pool of applicants (to minimise selection costs, which often far exceeds search costs, firms usually preselect a limited number of applicants) and rejects immediately all other applicants. The total number of matches in region j, M_j, is then equal to:

$$M_j = L_j v_j (1 - \exp^{-\sum_i k_{ij} L_i u_i / L_j v_j}). \tag{1}$$

So, the matching function M_j gives the number of preselected applicants at any one time as a function of the number of unemployed looking for jobs and the number of firms looking for workers in region i. This function is increasing in the number of unemployed and vacancies and has constant returns to scale.

The probability that a preselected applicant from region i is hired in region j is denoted as s_{ij}. Let q_{ij} denote the rate at which unemployed located in region i are hired in region j, and q_j denote the contact rate in region j. Let θ_{ij} denote the *(regional) labour market tightness,* the number of vacancies in region j relative to unemployment in region i. It follows that:

$$q_{ij} = s_{ij} . q_j , \tag{2}$$

where

$$q_j = \frac{M_j}{L_j v_j} = 1 - \exp^{-\sum_i k_{ij} \theta_{ij}^{-1}} \text{ and } \theta_{ij} = \frac{L_j v_j}{L_i u_i}. \tag{3}$$

The probability that an unemployed job seeker from region i is becoming employed in region j, depends on the preselection procedure of the firm. One possibility is that firms rank unemployed job seekers according to region and select the unemployed who lives closest to the firm. This behaviour can be shown to be optimal when job seekers are identical but for their commuting distance (and when employers have to partially compensate employees for the incurred commuting costs). Another possibility, which is more plausible, is that firms preselect randomly with respect to region. In the latter case, it follows that:

$$s_{ij} = \frac{A_{ij} k_{ij} u_i L_i}{\sum_{i'} k_{i'j} u_{i'} L_{i'}}, \tag{4}$$

where A_{ij} denotes the probability that firms which have established contact form a

[1] From a theoretical point of view, the assumption that search intensity is constant, and thus does not depend on the spatial labour market conditions is undesirable. However, empirical studies largely support the assumption that search intensity (of job seekers and firms) is relatively constant compared to (temporal and spatial) changes in the UV ratio (Russo, 1996; Burda and Profit, 1996).

match, which is either 0 or 1, so $\sum_i s_{ij} \le 1$. It is important to realise that A_{ij} is endogenously determined in the model.

The unemployment-employment transition rate
Given q_{ij} one can write f_{ij}, the rate at which unemployed job seekers in region i find employment in region j, as :

$$f_{ij} = \theta_{ij} q_{ij} . \tag{5}$$

So, the rate at which unemployed job seekers in region i find employment, f_i can be written as:[2]

$$f_i = \sum_j \theta_{ij} q_{ij} (\theta_{1j}^{-1}, .., \theta_{nj}^{-1}) . \tag{6}$$

2.2 Job destruction
It is presumed that jobs are destroyed at rate λ. In the present form, the destruction rate does not vary among regions.

2.3 Equilibrium Employment and Unemployment
In equilibrium, the number of individuals in region i which become unemployed must be equal to the number of individuals in region i which become employed. It is then straightforward to show that the *regional unemployment rate* u_i can be written as follows:

$$u_i = \frac{\lambda}{\lambda + \sum_j \theta_{ij} q_{ij}} . \tag{7}$$

Similarly, it can be shown that the *commuting share*, e_{ij}, defined as the number of jobs filled by residents from region j in region i, divided by the labour force in region i, can be written as:

$$e_{ij} = \frac{\theta_{ij} q_{ij}}{\lambda + \sum_j \theta_{ij} q_{ij}} . \tag{8}$$

2.4 Job creation
The value of a vacancy in region j, V_j, can be written as:

$$rV_j = -p_j c + \sum_i q_{ij} (J_{ij} - V_j) , \tag{9}$$

where $p_j c$ denotes the firms' search costs, which are presumed to be proportional to

[2] Given the properties of the contact function, it can be shown that f_{ij} is increasing in θ_{ij} Further, f_i is an increasing function of θ_{ij}, but is decreasing in $\theta_{i'j}$, $i' \neq i$.

regional productivity p_j, r denotes the discount rate, and J_{ij} denotes the value of a job filled by a worker living in region i. This expression can also be rewritten more conveniently as:

$$rV_j = -p_j c + q_{jf}(J_j^e - V_j), \tag{10}$$

where the expected value of a filled job J_j^e is defined as: $J_j^e = \frac{\sum_i J_{ij} s_{ij}}{\sum_i s_{ij}}$, and q_{jf} can be interpreted as the rate at which jobs are filled in region j. So, $q_{jf} = q_j(\theta_{1j},...,\theta_{nj})\sum_{i'} s_{i'j}$. The value of a job in region j occupied by a worker living in region i can be written as:

$$rJ_{ij} = p_j - w_{ij} - \lambda J_{ij} \text{ or, similarly, } J_{ij} = \frac{p_j - w_{ij}}{r+\lambda}, \tag{11}$$

where w_{ij} is the (bargained) wage paid by a firm in region j to a worker in region i. The expected value of a filled job can then be calculated as:

$$rJ_j^e = p_j - w_j^e - \lambda J_j^e, \tag{12}$$

where w_j^e denotes the expected wage level in region j, defined as: $w_j^e = \frac{\sum_{i'} w_{i'j} s_{i'j}}{\sum_{i'} s_{i'j}}$. In equilibrium, it is assumed that all profit opportunities from new jobs are exploited, driving rents from vacant jobs to zero. Therefore, the equilibrium condition for the supply of vacant jobs implies that $V_j = 0$. Consequently:

$$(r+\lambda)J^e = p_j - w_j^e = \frac{(r+\lambda)p_j c}{q_{jf}}. \tag{13}$$

This condition states that in equilibrium, regional labour market tightness is such that the expected value from a (new) job in region j is equal to the expected recruitment cost. So, the difference between the regional productivity and expected wage level is equal to the expected capitalised recruitment costs.

2.5 Workers

For unemployed workers, lifetime utility U_i can be written as

$$rU_i = z + \sum_j f_{ij}(W_{ij} - U_i), \tag{14}$$

where z denotes the utility of not working (mainly, the unemployment benefit) which is presumed to be the same in each region, and W_{ij} denotes the lifetime utility of a worker in region i which is employed in region j. For employed workers, lifetime utility can be written as

$$rW_{ij} = w_{ij} - t_{ij} + \lambda(U_i - W_{ij}) \qquad (15)$$

where t_{ij} denotes the generalised commuting costs (monetary and time costs) between i and j.

2.6 The Spatial Wage Equation

The wage rate is determined by a bargaining process between firms and preselected job applicants. The wage derived from the Nash bargaining solution is the wage that maximises the weighted product of the workers and firms net return from the job match (Pissarides, 2000). In order to form the job match, the worker gives up U_i for W_{ij}, and the firm gives up V_j for J_{ij}. Therefore the wage rate satisfies:

$$w_{ij} = \arg\max(W_{ij} - U_i)^{\beta}(J_{ij} - V_j)^{1-\beta}, \qquad (16)$$

where $0 \leq \beta \leq 1$. The parameter β may be interpreted as a relative measure of labour's bargaining strength, other than the one implied by the threat points U_i and V_j. For convenience, we will treat β as a parameter strictly between 0 and 1. Maximising the above equation with respect to the wage, renders the following first-order condition:

$$W_{ij} - U_i = \beta(J_{ij} - V_j + W_{ij} - U_i). \qquad (17)$$

So β can be interpreted as the worker' s share of the total surplus. By imposing the equilibrium condition $V_j = 0$, we obtain the following equation:

$$W_{ij} - U_i = \frac{\beta}{1-\beta} J_{ij}. \qquad (18)$$

So, the larger the firm's value of employing a worker from region i, J_{ij}, the larger the net value of employment for a worker from region i, $W_{ij} - U_i$. Using the above equations, it can be demonstrated that:

$$w_{ij} = (1-\beta)(t_{ij} + z) + \beta\sum_{j'} \theta_{ij'} s_{ij'} q_{j'} J_{ij} + \beta p_j, \qquad (19)$$

So, the wage is an increasing function of the commuting costs t_{ij} and the value of being unemployed, z. Since $1-\beta < 1$, this implies that the *net wage* (the wage minus the commuting costs) is a decreasing function of the commuting costs. Further, the wage is an increasing function of the productivity level in region j. The penultimate term can be interpreted as the average search/hiring costs per unemployed. Workers are rewarded for the saving of (future) hiring costs that the firm enjoys when a job is formed.[3] Given the wage rate w_{ij}, it is straightforward to calculate the expected wage level:

[3] This can be most easily seen when focusing on a one region economy. The wage rate can then be written as: $w = (1-\beta)(t + z) + \beta p(1 + c\frac{v}{u})$

$$w_j^e = (1-\beta)(t_j^e + z) + \beta \frac{\sum_{i'} s_{i'j} \sum_{j'} \theta_{i'j'} s_{i'j'} q_{j'} J_{i'j'}}{\sum_{i'} s_{i'j}} + \beta p_j, \tag{20}$$

where $t_j^e = \frac{\sum_{i'} t_{i'j} s_{i'j}}{\sum_{i'} s_{i'j}}$ denotes the expected commuting costs. Equation (20) implies that the regional productivity level has a positive effect on the wage level for three reasons. The first reason is that a higher productivity level increases the surplus. The second reason is, as we will show in the next section, that the expected commuting costs are an increasing function of the regional productivity level. In other words, workers are recruited from a larger range, which increases the commuting costs. The third reason is that the average wage costs are higher, since the ratio of vacancies to unemployment is less favourable increasing labour market bargaining position of the unemployed.

2.7 The reservation commuting costs

We have seen above that the wage is an increasing and the net wage a decreasing function of the commuting costs. It will be convenient to make this relationship explicit and we will write the wage rate as $w_{ij}(t_{ij})$ This implies that job seekers and firms will only form a match when the commuting costs are less than a certain maximum, which we will label the reservation commuting costs T_{ij} (this implies that the existence of the reservation commuting costs can be shown).

When J_{ij} is less than 0, W_{ij} -U_i is also less than 0, so firms and job seekers agree not to form a match. In contrast, when J_{ij} is more than 0, W_{ij} -U_i is also more than 0, so firms and job seekers agree to form a match. So, the reservation commuting costs T_{ij} is defined by the condition that J_{ij} is equal to 0. This condition implies that:

$$p_j - w_{ij}(T_{ij}) = 0. \tag{21}$$

and using the wage equation, the reservation commuting costs can be written as:

$$T_{ij} = p_j - z - \frac{\beta}{1-\beta} \sum_{j'} \theta_{ij'} s_{ij'} q_{j'} J_{ij'}. \tag{22}$$

2.8 The probability of a match after a job applicant has been preselected

The conditional probability of a match, which had been defined as A_{ij}, in section 2.1, is equal to 1 when $T_{ij} > t_{ij}$ and 0 otherwise.

3. Equilibrium

On the basis of the above components, the model can be summarised. For a full list of variables, see next section. Labour market tightness is defined as:

$$\theta_{ij} = \frac{L_j v_j}{L_i u_i}.$$

In the steady state equilibrium, the following flow employment and unemployment equilibrium conditions hold:

$$e_{ij} = \frac{\theta_{ij} q_{ij}}{\lambda + \sum_{j'} \theta_{ij'} q_{ij'}}.$$

$$u_i = \frac{\lambda}{\lambda + \sum_{j'} \theta_{ij'} q_{ij'}}.$$

The job creation condition:

$$p_j - w_j^e = \frac{(r+\lambda) p_j c}{q_{jf}}.$$

The expected wage equation:

$$w_j^e = (1-\beta)(t_j^e + z) + \beta \frac{\sum_{i'} s_{i'j} \sum_{j'} \theta_{i'j'} s_{i'j'} q_{j'} J_{i'j'}}{\sum_{i'} s_{i'j}} + \beta p_j.$$

The expected commuting costs:

$$t_j^e = \frac{\sum_{i'} t_{i'j} s_{i'j}}{\sum_{i'} s_{i'j}}.$$

The value of a filled job:

$$J_{ij} = \frac{p_j - w_{ij}}{r+\lambda}.$$

The wage rate w_{ij} is defined as:

$$w_{ij} = (1-\beta)(t_{ij} + z) + \beta \sum_{j'} \theta_{ij'} s_{ij'} q_{j'} J_{ij} + \beta p_j.$$

The reservation commuting distance/costs:

$$T_{ij} = p_j - z - \frac{\beta}{1-\beta} \sum_{j'} \theta_{ij'} s_{ij'} q_{j'} J_{ij'}.$$

Given information on the reservation commuting distance/costs, one can calculate the probability of filling a vacancy, A_{ij}:

A_{ij} is equal to 1 when $T_{ij} > t_{ij}$ and 0 otherwise.

Given information on the probability of filling a vacancy given a preselection, we get the probability that a vacancy in region j is filled by a job seeker in region i:

$$s_{ij} = \frac{A_{ij} k_{ij} u_i L_i}{\sum_{i'} k_{i'j} u_{i'} L_{i'}}.$$

Further, we use the following equation that determines the rate at which unemployed located in region i are hired in region j:

$$q_{ij} = s_{ij} . q_j ,$$

The rate at which jobs are filled in region j:

$$q_{jf} = q_j(\theta_{1j},...,\theta_{nj}) \sum_{i'} s_{i'j} .$$

Finally, we need an equation for the contact rate for vacancy in region j:

$$q_j = 1 - \exp^{-\sum_i k_{ij}\theta_{ij}^{-1}}$$

The number of regions in the economy is assumed to be equal to n. So the above model consist of $8n^2$ and 6n equations. The corresponding $8n^2$ and 6n endogenous variables have been listed in the appendix, where also the exogenous variables can be found.

4. Discussion

In this paper, we have described the functioning of a regional equilibrium model for the labour market. Regional labour supply is fixed, but vacancies, unemployment and wage levels are endogenously determined at the regional level given regional variation in productivity levels. One of the strengths of the model is that the (maximum) commuting costs are also endogenously determined, and depend, for example, on the regional productivity level. Another strength of the model is that regional unemployment and vacancy levels are never zero, due to search frictions . The main weakness of the model is that migration between regions and regional variation in price levels of land (for production and housing) are ignored. These issues will be addressed in another paper.

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Appendix 1: List of Endogenous and Exogenous Variables

Endogenous Variables:

q_j the rate at which unemployed are contacted in region j.

A_{ij} the probability that an applicant from region i is hired in region j, conditional on a contact between job seeker from i and region j

s_{ij} the probability that an applicant from region i is hired in region j given that the firm in region j is contacted

T_{ij} reservation commuting costs

θ_{ij} the ratio of the number of vacancies in region j to the number of unemployed in region i, referred to as regional labour market tightness

J_{ij} the value of a job in region j

u_i the unemployment rate in region i

v_j the vacancy rate (in terms of employment) in region j

q_{ij} the rate at which unemployed located in region i are hired in region j

q_{jf} the rate at which vacancies are filled in region j

e_{ij} the share of jobs in region i filled by the unemployed from region j

w_{ij} the wage earned in region j by residents from region i

w_j^e the expected wage in region j

t_j^e the expected commuting costs in region j

Exogenous Variables:

λ the rate at which jobs are destroyed

z the utility of unemployment (unemployment benefits)

c the firms' search costs relative to the productivity level

t_{ij} the commuting costs

k_{ij} the probability that an unemployed worker living in region i applies for a vacancy in region j

p_j the regional productivity level in region j

L_i the labour force in region i

r the discount rate

β the relative measure of labour's bargaining strength